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Models of neutrino masses from oscillations with maximal mixing

Guido Altarelli

Theoretical Physics Division, CERN,
CH - 1211 Geneva 23 and
Università di Roma Tre, Rome, Italy
E-mail: Guido.Altarelli@cern.ch

Ferruccio Feruglio

Università di Padova and I.N.F.N., Sezione di Padova,
Padua, Italy
E-mail: ferruccio.feruglio@pd.infn.it

Abstract: We discuss models of neutrino masses that lead to a large mixing angle for atmospheric neutrino oscillations. In particular we study a mechanism where a simple texture for the Dirac matrix leads to the observed pattern of mixings for a continuous range of Majorana matrices in a reasonably natural way. Both nearly maximal and small mixing solutions for solar neutrinos are compatible with this model. Possible dynamical realizations are discussed and a detailed example in terms of horizontal U(1) charges is presented.

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Following the experimental results from Superkamiokande [1] a lot of attention has been devoted to the problem of a natural explanation of the observed nearly maximal mixing angle for atmospheric neutrino oscillations. It is possible that also solar neutrino oscillations, if explained by vacuum oscillations, occur with a large mixing angle [2]. Large mixing angles are somewhat unexpected because the observed quark mixings are small and the quark, charged lepton and neutrino mass matrices are to some extent related in Grand Unified Theories. In a previous paper [3] we have given a first discussion of this problem. Within the framework of the see-saw mechanism we focussed on the interplay between the neutrino Dirac and Majorana matrices which is necessary in order to generate maximal mixing [4]. In the present note we continue the study of this issue. We start by giving a general formulation of the problem and by specifying our working assumptions. Among our previously proposed strategies for a natural explanation of maximal mixing, we select and further elaborate on one which we find particularly plausible and appealing. For our mechanism to work we need that, in analogy with what is observed for quark and leptons, one mass eigenvalue is dominant for neutrinos (for both the Dirac and the effective light neutrino mass matrices). Also, in zeroth approximation, we need that the neutrino Dirac mass matrix, in the basis where charged leptons are diagonal, has only two large entries of comparable magnitude. We then show that either single or double nearly maximal mixing occur automatically for whatever Majorana mass matrix chosen in a large domain of parameter space, without need of fine tuning. We discuss models where these conditions can be realized and give numerical examples that fit the present data. We finally compare ours with other possible mechanisms.

We start by assuming only three flavours of neutrinos that receive masses from the see-saw mechanism [5] and allow all possible hierarchical patterns for the neutrino mass eigenvalues m_i. For reasons of simplicity, we consider the simplest version of the see-saw mechanism with one Dirac, m_D, and one Majorana, M, mass matrix, related to the neutrino mass matrix m_ν, in the basis where the charged lepton mass matrix is diagonal, by

$$m_\nu = m_D^T M^{-1} m_D \,. \tag{1}$$

As well known this is not the most general see-saw mechanism because we are not including the left-left Majorana mass block. Maximal atmospheric neutrino mixing and the requirement that the electron neutrino does not participate in the atmospheric oscillations, as indicated by the Superkamiokande [1] and Chooz [6] data, lead directly to the following structure of the U_{fi} (f=e,μ,τ, i=1,2,3) real orthogonal mixing matrix, apart from sign convention redefinitions (here we are not interested in CP violation effects: all matrices are taken real)

$$U_{fi} = \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & +1/\sqrt{2} \end{bmatrix} . \tag{2}$$

This result is obtained by a simple generalization of the analysis of ref. [7] (also discussed in ref. [8]) to the case of arbitrary solar mixing angle ($s \equiv \sin\theta_{sun}$, $c \equiv \cos\theta_{sun}$): $c = s = 1/\sqrt{2}$ for maximal solar mixing (e.g. for vacuum oscillations $\sin^2 2\theta_{sun} \sim 0.75$), while $\sin^2 2\theta_{sun} \sim 4s^2 \sim 5.5 \cdot 10^{-3}$ for the small angle MSW [9] solution. The vanishing of U_{e3} guarantees that ν_e does not participate in the atmospheric oscillations and the relation $|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$ implies maximal mixing for atmospheric neutrinos. Note that we are assuming only two frequencies, given by

$$\Delta_{sun} \propto m_2^2 - m_1^2\,, \qquad \Delta_{atm} \propto m_3^2 - m_{1,2}^2\,. \tag{3}$$

The effective light neutrino mass matrix is given by $m_\nu = U m_{diag} U^T$ with $m_{diag} = Diag[m_1, m_2, m_3]$. For generic s one finds

$$m_\nu = \begin{bmatrix} 2\epsilon & \delta & \delta \\ \delta & \frac{m_3}{2} + \epsilon_2 & -\frac{m_3}{2} + \epsilon_2 \\ \delta & -\frac{m_3}{2} + \epsilon_2 & \frac{m_3}{2} + \epsilon_2 \end{bmatrix}, \tag{4}$$

with

$$\epsilon = (m_1 c^2 + m_2 s^2)/2\,, \qquad \delta = (m_1 - m_2) cs/\sqrt{2}\,, \qquad \epsilon_2 = (m_1 s^2 + m_2 c^2)/2\,. \tag{5}$$

With respect to the notation of ref. [3] we have reinstated the normalization factor $m_3/2$ in m_ν. We see that the existence of one maximal mixing and $U_{e3} = 0$ are the most important input that leads to the matrix form in eq. (4), (5). The value of the solar neutrino mixing angle can be left free. While the simple parametrization of the matrix U in eq. (2) is quite useful to guide the search for a realistic pattern of neutrino mass matrices, it should not be taken too literally. In particular the data do not exclude a non-vanishing U_{e3} element. In most of the Superkamiokande allowed region the bound by Chooz [6] amounts to $|U_{e3}| \lesssim 0.2$. In the region not covered by Chooz $|U_{e3}|$ can even be larger [10, 11]. Thus neglecting $|U_{e3}|$ with respect to s in eq. (2) is not really justified. Also note that in presence of a large hierarchy $|m_3| \gg |m_{1,2}|$ the effect of neglected parameters in eq. (4) can be enhanced by $m_3/m_{1,2}$ and produce seizable corrections. A non vanishing U_{e3} term can lead to different $(m_\nu)_{12}$ and $(m_\nu)_{13}$ terms. Similarly a deviation from maximal mixing $U_{\mu 3} \neq U_{\tau 3}$ distorts the ϵ_2 terms in the 23 sector of m_ν. Therefore, especially for a large hierarchy, there is more freedom in the small terms in order to construct a model that fits the data than it is apparent from eq. (4).

Given the observed frequencies and our notation in eq. (3), there are three possible hierarchies of mass eigenvalues:

$$\begin{aligned} &\mathrm{A} : |m_3| >> |m_{2,1}|\,, \\ &\mathrm{B} : |m_1| \sim |m_2| >> |m_3|\,, \\ &\mathrm{C} : |m_1| \sim |m_2| \sim |m_3| \end{aligned} \tag{6}$$

(in case A there is no prejudice on the m_1, m_2 relation). For B and C different subcases are then generated according to the relative sign assignments for $m_{1,2,3}$. For each case we can set to zero the small masses and mixing angles and find the effective light neutrino matrices which are obtained both for double and single maximal mixing. Note that here we are working in the basis where the charged lepton masses are diagonal, and approximately given by $m_l = Diag[0,0,m_\tau]$. For model building one has to arrange both the charged lepton and the neutrino mass matrices so that the neutrino results coincide with those given here after diagonalization of charged leptons. For example, in case A, $m_{diag} = Diag[0,0,m_3]$ and we obtain

$$m_\nu/m_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}. \tag{7}$$

In this particular case the results are the same for double and single maximal mixing. Note that the signs correspond to the phase convention adopted in eq. (2). If one prefers all signs to be positive it is sufficient to invert the sign of the third row of the matrix U in eq. (2). We can similarly proceed in the other cases and we obtain the results in table 1 (where the overall mass scale was dropped), which we now discuss.

The completely degenerate case C is the only one that could in principle accommodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. For this the common mass should be around 1-3 eV. Then the solar frequency could be given by a small 1-2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2-3 splitting. Note that we have not included in Table 1 the degenerate case C with three equal signs. Clearly the mixing matrix in this case is completely determined by the small degeneracy splitting corrections and the zeroth order approximation is irrelevant. A strong constraint arises in the degenerate case from neutrinoless double beta decay which requires that the ee entry of m_ν must obey $|(m_\nu)_{ee}| \leq 0.46$ eV [12]. As observed in ref. [13], this bound can only be satisfied if bimixing (that is double maximal mixing) is realized. In fact we see from the expression of ϵ in eq. (5) that we need $m_1 \sim -m_2$ and $c^2 \sim s^2$ for a cancellation to occur in $(m_\nu)_{ee}$. Since the solar mixing can only be either near maximal or very small, only the bimixing solutions C1 and C2 survive (which are physically equivalent [1]), as can be verified in table 1. So all other solutions of type C can only survive if the common mass is below 0.46 eV. We think that it is not at all clear at the moment that a hot dark matter component is really needed [14]. However the main reason to consider the fully degenerate solution is that it is compatible with hot dark matter, so that only the solution C1/C2 in the case of bimixing is interesting. Note that for degenerate masses with $m \sim 1-3$ eV we need a relative splitting $\Delta m/m \sim \Delta m^2_{atm}/2m^2 \sim 10^{-3} - 10^{-4}$ and a much smaller one for solar neutrinos explained by vacuum oscillations: $\Delta m/m \sim 10^{-10} - 10^{-11}$. We are unable to imagine a natural mechanism compatible with unification and the see-saw mechanism leading to such a precise near symmetry. An ansatz of this sort has been proposed in ref. [15].

[1] The solutions C1 and C2 are related by the exchange $s \leftrightarrow c$.

	m_{diag}	double maximal mixing	single maximal mixing
A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
C3	Diag[1,1,-1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Table 1: Zeroth order form of the neutrino mass matrix for double and single maximal mixing, according to the different possible hierarchies given in eq. (6).

Of the remaining possibilities two are particularly remarkable for their simplicity: solution A (eq. (7)) and solution B1 (for bimixing). Of the six independent entries of a symmetric matrix three (four) are zero for solution A (B1 bimixing). However, solution A requires the entries in the 23 sector to be related (in particular the 23 subdeterminant must be zero, in order for two of the eigenvalues to be vanishing in zeroth order, as assumed in this case), solution B1 requires the two non zero entries in the first row or column to be nearly equal. In the following we shall discuss how these patterns can be generated in a natural way.

As a first orientation we observe that, after diagonalization of the charged lepton Dirac mass matrix, we still have the freedom of a change of basis for the right-handed neutrino fields. In fact right-handed charged lepton and neutrino fields, as opposed to left-handed fields, are uncorrelated by $SU(2) \otimes U(1)$ gauge symmetry. We can use

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this freedom to make the Majorana matrix diagonal: $M^{-1} = O^T d_M O$ with $d_M = Diag[1/r_1, 1/r_2, 1/r_3]$. Then if we parametrize the matrix Om_D by z_{ab} we have:

$$m_{\nu ab} = (m_D^T M^{-1} m_D)_{ab} = \sum_c \frac{z_{ca} z_{cb}}{r_c} . \tag{8}$$

From this expression we see that, while we can always arrange the twelve parameters z_{ab} and r_a to arbitrarily fix the six independent matrix elements of m_ν, case A is special in that it can be approximately reproduced in two particularly simple ways, without relying on precise cancellations among different terms:
i) One of the right-handed neutrinos is particularly light and, in first approximation, it is only coupled to μ and τ [16]. Thus, $r_c \sim \eta$ (small) and $z_{c1} \sim 0$. In this case [11, 17] the 23 subdeterminant vanishes, and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1.
ii) There are only two large entries in the z matrix, $|z_{c2}| \sim |z_{c3}|$, and the three eigenvalues r_a are of comparable magnitude (or, at least, with a less pronounced hierarchy than for the z matrix elements). Then, again, the subdeterminant 23 vanishes and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1.

The possibility ii), which was noticed in our previous paper [3], involves no more fine tuning than i). One solution is more specific on M, the other on m_D. Note that solution B1 requires a more delicate balance between the Dirac and Majorana matrices, which, however, can be realized in particular models [11, 17], as we shall see in the following.

We now discuss the mechanism ii) in more detail. We start with the order zero approximation, as in eq. (7) or table 1, where all small entries in the matrix are set to zero. Assume that, in the basis where charged leptons are diagonal, the Dirac matrix m_D, defined by the bilinear $\bar{\psi}_R m_D \psi_L$, takes the approximate form

$$m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{bmatrix} . \tag{9}$$

This matrix has the property that for a generic Majorana matrix M one finds

$$m_\nu = m_D^T M^{-1} m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{bmatrix} . \tag{10}$$

The only condition on M^{-1} is that the 33 entry is non zero. The reason for this insensitivity to M is that m_D given by eq. (9) can be diagonalized by a transformation of the form $V^T m_D U = d \propto Diag[0, 0, 1]$ with V (the right-handed field transformation) given by a block-diagonal matrix with $(V)_{33} = 1$ and $(V)_{13} = (V)_{31} = (V)_{23} = (V)_{32} = 0$. Then we can write $m_\nu = U d V^T M^{-1} V d U^T$. The diagonal matrix d acts like a projector so that only the 33 entry of $dV^T M^{-1} V d$ is non-vanishing. Here it is crucial that the neutrino eigenvalues are approximately $(0, 0, m_3)$. Finally the U transformation (the left-handed field rotation) brings this matrix into the form of eq. (10) that for $|x| -> 1$ approaches that required by solution A. Note that the 23 subdeterminant of m_ν is vanishing for all values of x. The matrix U is directly the neutrino mixing matrix. Its form is

$$U = \begin{bmatrix} c & -s & 0 \\ sc_\gamma & cc_\gamma & -s_\gamma \\ ss_\gamma & cs_\gamma & c_\gamma \end{bmatrix}, \tag{11}$$

with

$$s_\gamma = -x/r\,, \qquad c_\gamma = 1/r\,, \qquad r = \sqrt{1+x^2}\,. \tag{12}$$

We see that at $|x| \neq 1$, U_{e3} still vanishes but the atmospheric neutrino mixing is no more maximal. Rather one has

$$\sin^2 2\theta = 4s_\gamma^2 c_\gamma^2 = \frac{4x^2}{(1+x^2)^2}\,. \tag{13}$$

Thus the bound $\sin^2 2\theta \gtrsim 0.8$ translates into $0.6 \lesssim |x| \lesssim 1.6$. It is interesting to recall that in ref. [18] it was shown that the mixing angle can be amplified by the running from a large mass scale down to low energy.

A zeroth order texture as in eq. (9) with generic entries $(m_D)_{23} = A$ and $(m_D)_{33} = B$ can well apply for both charged leptons and neutrinos. If for charged leptons we start with A_l and B_l and for neutrinos with A_ν and B_ν, after diagonalisation of the charged leptons by a left-handed matrix of the form in eq. (11) with $s = 0$, one obtains a matrix of the same form with $A/B = x$, as required for m_D in eq. (9). Precisely the value of $|x|$ is given by $|x| = |(A_l B_\nu - B_l A_\nu)/(A_l A_\nu + B_l B_\nu)|$.

It remains for us to discuss if the proposed strategy is not completely ruined by going beyond the zeroth order approximation. In fact terms of order λ, λ^2 ... have to be added to m_D to introduce a hierarchical structure as observed for quarks and leptons. Since, in general, also M will be hierarchical in terms of some other small parameter η, M^{-1} can contain terms of order $1/\eta$, $1/\eta^2$..... In the see-saw product, the ratios of a power of λ and a power of η can generate unwanted additional terms of order one or even larger that can spoil the zeroth order result. Obviously a necessary condition is that the Majorana hierarchy is not too strong in comparison of the Dirac hierarchy, or that η is not smaller than some power of λ. We discuss this issue by showing that reasonable models exist where the mechanism is indeed realised.

An attractive approach to reproduce the observed features of quark and lepton mass matrices is in terms of horizontal symmetries, in particular, for simplicity, U(1) charges [19]. When the charges do not match by n the corresponding mass term is suppressed by n powers of a small parameter that arises from the vacuum expectation value (divided by some large mass) of one or more extra scalar fields θ_i that carry one unit of charge. There can be two different fields with opposite charges $\theta_\pm$, [2] so that the suppression factors can involve two different small parameters, according to the sign of n. We now show that our mechanism can be realised in a simple U(1) model with two oppositely charged fields $\theta_\pm$.

[2]We regard the conjugate field θ^* as distinct from θ. Therefore θ_- may coincide with $(\theta_+)^*$.

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We want to construct an example of charge assignments that lead to the desired form for the Dirac and Majorana matrices. In particular, we demand that, for the Majorana matrix M, the determinant is of order 1 (because we want to avoid that one lightest eigenvalue exists, which would be typical of mechanism i), while our case prefers comparable eigenvalues) and the same is true for the matrix element $(M^{-1})_{33}$ (which plays a special role in our mechanism). One possibility is to start with the following charges for the lepton doublet L and the right-handed neutrino fields:

$$L = (2, 0, 0)\,; \qquad \bar{R}_\nu = (1, -1, 0) \tag{14}$$

which lead to the textures

$$m_D \propto \begin{bmatrix} \lambda^3 & \lambda & \lambda \\ \lambda & \lambda' & \lambda' \\ \lambda^2 & 1 & 1 \end{bmatrix}; \qquad M \propto \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}. \tag{15}$$

It is simple to verify that, for generic coefficients, the effective light neutrino matrix is of the form

$$m_\nu \propto \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}. \tag{16}$$

However, the crucial property is that the 23 subdeterminant of m_ν automatically vanishes in the limit $\lambda \to 0$, because of the simple mathematical property displayed by eqs. (9), (10). Actually, for generic coefficients, the 23 minor is of order $\lambda\lambda'$. We now specify the charges of right-handed charged leptons. The requirements are that the hierarchy of eigenvalues corresponds to the observed e, μ, τ mass ratios and that after diagonalisation of the charged leptons the form of the Dirac matrix for neutrinos is not spoiled (for this we need that the left-handed diagonalising matrix is of the form in eq. (11) with s small). An acceptable choice for the right-handed lepton charges is

$$\bar{R}_l = (-4, 1, 0) \tag{17}$$

which leads to

$$m_l \propto \begin{bmatrix} \lambda'^2 & \lambda'^4 & \lambda'^4 \\ \lambda^3 & \lambda & \lambda \\ \lambda^2 & 1 & 1 \end{bmatrix}. \tag{18}$$

This matrix has eigenvalues of order $(\lambda'^2, \lambda, 1)$. For $\lambda \sim \lambda' \sim 10^{-1} - 10^{-2}$ and generic coefficients one obtains numerically reasonable solutions. In this analysis we have also taken into account the most general kinetic terms compatible with the chosen charge assignement. We found that numerically acceptable solutions are generated even in the presence of these terms. (See also ref. [21]). Most of them are of the bimixing type. This is because, as seen from eq. (16), the 11 entry of m_ν is much smaller than the 12, 13 entries. Given the general expression eq. (4) this particular pattern corresponds to double nearly maximal mixing.

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It is interesting to note that if all the $U(1)$ charges for the left-handed lepton doublets and the left-handed anti-neutrinos have the same sign, the relation between the Dirac and Majorana hierarchies is too tight for the mechanism to work. In fact in this case, the neutrino Dirac and Majorana mass matrices can be written as [20] $m^\nu_D = Q_{\nu\bar{R}} Y_D Q_L$ and $M = Q_{\nu\bar{R}} Y_M Q_{\nu\bar{R}}$ where the Q's are diagonal matrices of the form $Q = \mathrm{Diag}[\lambda^{q_1}, \lambda^{q_2}, \lambda^{q_3}]$ in terms of one single small parameter λ and the charges q_i. Y are generic non hierarchical Yukawa matrices. It is easy to see that in constructing m_ν the right-handed charges drop away and $m_\nu = Q_L Y_D^T Y_M^{-1} Y_D Q_L$. With the above left-handed charge assignments the resulting matrix m_ν indeed has the 22, 23, 32, 33 entries of order 1 while all others are suppressed, but the 23 subdeterminant is not vanishing because the matrix $Y_D^T Y_M^{-1} Y_D$ is completely generic. Thus the resulting eigenvalue structure is not the right one in general and must be fixed by hand. It is easy to see why our mechanism does not work in this case. In fact, the same reason that ensures the cancellation of right-handed charges in the see-saw product makes the connection between the hierarchies of the Dirac and Majorana matrices such that the λ terms of m_D and the $1/\lambda$ terms of M^{-1} necessarily generate finite terms that spoil the vanishing of the 23 subdeterminant. So models of type A cannot in general be explained in terms of all positive (or all negative) L and $\bar{R}_\nu$ charges. For our mechanism to work we need that the Dirac and Majorana mass matrices are generated by less tightly connected terms, as in the above example. This could also be the case in models based on GUT's [22], for example Susy $SO(10)$ or flipped $SU(5)$, where m_D and M are generated by different Higgs multiplets and representations. Note that the model B1 (bimixing) can be reproduced in terms of left-handed charges (a,-a,-a) for (e_L, μ_L, τ_L), as observed in ref. [11, 17], but the 12 and 13 entries, of order 1, must be arranged to be nearly equal.

In conclusion, we have studied a mechanism, introduced in [3], that can explain the observed pattern of neutrino oscillations, both for double and single maximal mixing. In our case both for Dirac and light neutrino masses there is one dominant eigenvalue: $|m_3| >> |m_{1,2}|$ as in case A of table 1. Our proposed mechanism only contains a minimum of fine tuning and compares well with all other mechanisms sofar presented [22, 23, 24, 25, 26]. Some of them were also discussed here. From these examples we learn that after all large mixings are not so unplausible as it appears at first sight. In all of these models there is no reason why the atmospheric neutrino mixing should be maximal but it can be large. In many cases once the atmospheric mixing is large also the solar mixing turns out to be large, but this is not necessarily true in all models, for example in our model.

References

[1] Y. Fukuda et al., *Phys. Lett.* **B 436** (1998) 33 [hep-ex/9805006]; *Phys. Rev. Lett.* **81** (1998) 1158, Erratum-ibid. **81** (1998) 4279 [hep-ex/9805021]; *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003].

[2] P. Osland and G. Vigdel, *Phys. Lett.* **B 438** (1998) 129 [hep-ph/9806339];
J.N. Bahcall, P.I. Krastev and A. Yu Smirnov, *Phys. Rev.* **D 58** (1998) 096016 [hep-ph/9807216].

[3] G. Altarelli and F. Feruglio, *Phys. Lett.* **B 439** (1998) 112 [hep-ph/9807353].

[4] For a similar approach, see also M. Jezabek and Y. Sumino, hep-ph/9807310.

[5] M. Gell-Mann, P. Ramond and R. Slansky in Supergravity, edited by P. van Nieuwenhuizen and D. Z. Freedman, North-Holland, Amsterdam, 1979, p.315;
T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto, KEK report No. 79-18, Tsukuba, Japan, 1979.

[6] M. Apollonio et al., *Phys. Lett.* **B 420** (1998) 397 [hep-ex/9711002].

[7] V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, *Phys. Lett.* **B 437** (1998) 107 [hep-ph/9806387];
V. Barger, T.J. Weiler and K. Whisnant, hep-ph/9807319.

[8] S.M. Bilenky and C. Giunti, hep-ph/9802201;
A.J. Baltz, A.S. Goldhaber and M. Goldhaber, hep-ph/9806540.

[9] L. Wolfenstein, *Phys. Rev.* **D 17** (1978) 2369;
S.P. Mikheyev and A. Yu Smirnov, *Sov. J. Nucl. Phys.* **42** (1986) 913.

[10] M.C. Gonzalez-Garcia, H. Nunokawa, O.L.G. Peres and J.W.F. Valle, hep-ph/9807305;
G.L. Fogli, E. Lisi, A. Marrone and G. Scioscia, hep-ph/9808205;
S.M. Bilenkii, C. Giunti and W. Grimus, hep-ph/9809368.

[11] R. Barbieri, L.J. Hall, D. Smith, A. Strumia and N. Weiner, hep-ph/9807235.

[12] L. Baudis et al., *Phys. Lett.* **B 407** (1997) 219.

[13] F. Vissani, hep-ph/9708483;
H. Georgi and S.L. Glashow, hep-ph/9808293.

[14] See for example L.K. Krauss, hep-ph/9807376.

[15] M. Fukugita, M. Tanimoto and T. Yanagida, *Phys. Rev.* **D 57** (1998) 4429 [hep-ph/9709388];
M. Tanimoto, hep-ph/9807283 and hep-ph/9807517;
H. Fritzsch and Z. Xing, *Phys. Lett.* **B 440** (1998) 313 [hep-ph/9808272];
R.N. Mohapatra and S. Nussinov, hep-ph/9808301 and hep-ph/9809415;
M. Fukugita, M. Tanimoto and T. Yanagida, hep-ph/9809554.

[16] S.F. King, hep-ph/9806440;
S. Davidson and S.F. King, hep-ph/9808296.

[17] R. Barbieri, L.J. Hall and A. Strumia, hep-ph/9808333.

[18] M. Tanimoto, *Phys. Lett.* **B 360** (1995) 41 [hep-ph/9508247];
G.K. Leontaris, S. Lola and G.G. Ross, *Nucl. Phys.* **B 454** (1995) 25 [hep-ph/9505402];
G.K. Leontaris, S. Lola, C. Scheich and J.D. Vergados, *Phys. Rev.* **D 53** (1996) 6381 [hep-ph/9509351];
J. Ellis, G.K. Leontaris, S. Lola and D.V. Nanopoulos, hep-ph/9808251.

[19] C. Froggatt and H.B. Nielsen, *Nucl. Phys.* **B 147** (1979) 277.

[20] A. Rasin and J.P. Silva, *Phys. Rev.* **D 49** (1994) 20 [hep-ph/9309240];
P. Ramond, hep-ph/9809401.

[21] M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B 420** (1994) 468 [hep-ph/9310320].

[22] K.S. Babu and S.M. Barr, *Phys. Lett.* **B 381** (1996) 202 [hep-ph/9511446];
Z. Berezhiani and Z. Tavartkiladze, *Phys. Lett.* **B 396** (1997) 150 [hep-ph/9612277]; *Phys. Lett.* **B 409** (1997) 220 [hep-ph/9612232];
G. Costa and E. Lunghi, *Nuovo Cim.* **A 110** (1997) 549 [hep-ph/9709271];
B. Brahmachari and R.N. Mohapatra, *Phys. Rev.* **D 58** (1998) [hep-ph/9710371];
A.S. Joshipura and A.Yu. Smirnov, *Phys. Lett.* **B 439** (1998) 103 [hep-ph/9806376];
J.C. Pati, hep-ph/9807315;
I.S. Sogami, H. Tanaka and T. Shinohara, hep-ph/9807449;
Y. Nomura and T. Yanagida, hep-ph/9807325;
K. Oda, E. Takasugi, M. Tanaka and M. Yoshimura, hp-ph/9808241;
J. Sato and T. Yanagida, hep-ph/9809307;
W. Buchmüller and T. Yanagida, hep-ph/9810308.

[23] Y. Chikira, N. Haba and Y. Mimura, hep-ph/9808254;
S. Mohanty and D.P. Roy, hep-ph/9808451;
E. Ma, D.P. Roy and U. Sarkar, hep-ph/9810309.

[24] M. Drees, S. Pakvasa, X. Tata and T. ter Veldhuis, *Phys. Rev.* **D 57** (1998) 5335 [hep-ph/9712392];
E.J. Chun, S.K. Kang, C.W. Kim and U.W. Lee, hep-ph/9807327;
V. Bednyakov, A. Faessler and S. Kovalenko, hep-ph/9808224;
A.S. Joshipura and S.K. Vempati, hep-ph/9808232;
B. Mukhopadhyaya, S. Roy and F. Vissani, hep-ph/9808265;
O.C.W. Kong, hep-ph/9808304.

[25] P. Binetruy, S. Lavignac, S. Petcov and P. Ramond, *Nucl. Phys.* **B 496** (1997) 3 [hep-ph/9610481];
N. Irges, S. Lavignac and P. Ramond, *Phys. Rev.* **D 58** (1998) 5003 [hep-ph/9802334];
J.K. Elwood, N. Irges and P. Ramond, *Phys. Rev. Lett.* **81** (1998) 5060 [hep-ph/9807228];
P. Ramond, hep-ph/9808489;
Y. Grossman, Y. Nir and Y. Shadmi, *JHEP* **10** (1998) 007 [hep-ph/9808355].

[26] Z.G. Berezhiani and A. Rossi, *Phys. Lett.* **B 367** (1996) 219 [hep-ph/9507393];
E. Torrente-Lujan, *Phys. Lett.* **B 389** (1996) 557 [hep-ph/9604218];
M. Bando, T. Kugo and K. Yoshioka, *Phys. Rev. Lett.* **80** (1998) 3004 [hep-ph/9710417];
B.C. Allanach, hep-ph/9806294;
R.P. Thun and S. McKee, *Phys. Lett.* **B 439** (1998) 123 [hep-ph/9806534];
D.V. Ahluwalia, *Mod. Phys. Lett.* **A 13** (1998) 2249 [hep-ph/9807267];
G. Eyal, hep-ph/9807308;
E. Ma, hep-ph/9807386;
N. Haba, hep-ph/9807552.

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